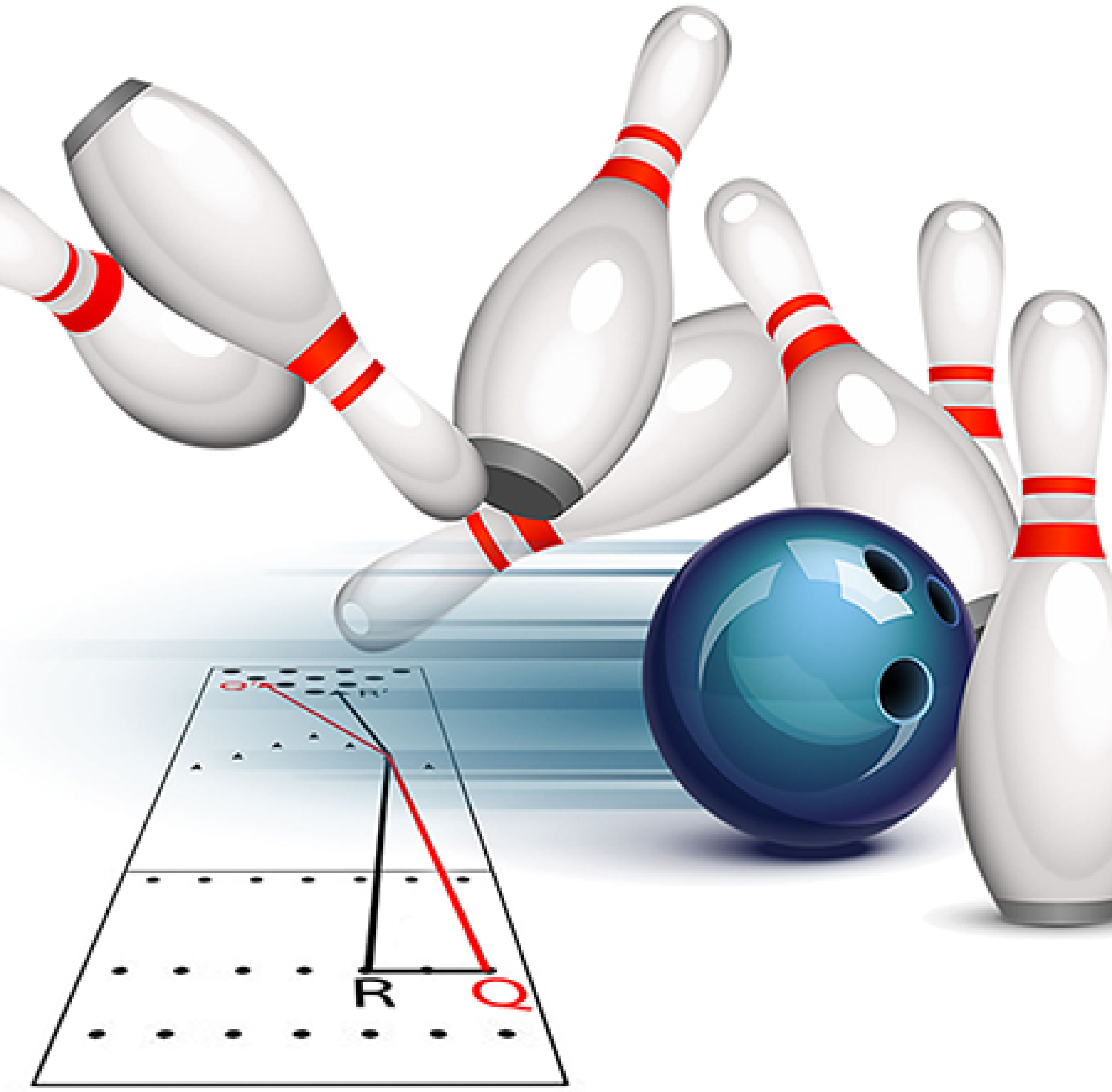
BOWLING AND MATHEMATICS



By Marvin L. Bittinger

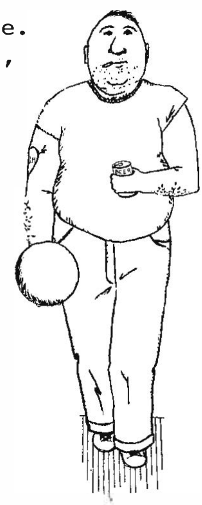
Bowling and Mathematics

Marvin L. Bittinger

What is your image of a bowler? Is it similar to the person in Figure 1?

<u>Typical Mannerisms and Thoughts</u>

1. Where's the beer?
2. Who's collecting the pot?
3. Doesn't believe in practice.
4. Uses one ball, 90 hardness, and throws it across the 4th arrow.
5. Has triplicate 158 patch.

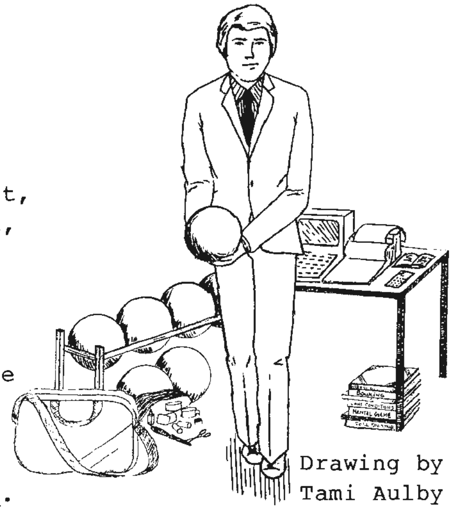


Drawing by
Tami Aulby

Or, is your image of a bowler similar to the person in Figure 2?

Typical Mannerisms and Thoughts

1. Where's the oil?
2. Where's the shot?
3. Which of the eight balls that I have should I use?
4. Should I use top weight, finger weight, positive weight, negative weight, thumb weight, or bottom weight?
5. Should I use conventional, semi-conventional, or finger-tip grip?
6. Always practices at least once a week.
7. What spare conversion system should I use?
8. Always reads Bowler's Journal.



Drawing by Tami Aulby

Unfortunately, most people have an image of a bowler that fits quite closely with the person in Figure 1. In truth, there are many aspects of the person in Figure 2 in most good bowlers. I have spent a great deal of time observing and talking to professional bowlers during the past few years, and whatever they are, they are NOT dumb. In fact, the better you become, the more mental the game becomes. George Allen, in his recent book, The Mental Game, makes this point by using a graph like the following:

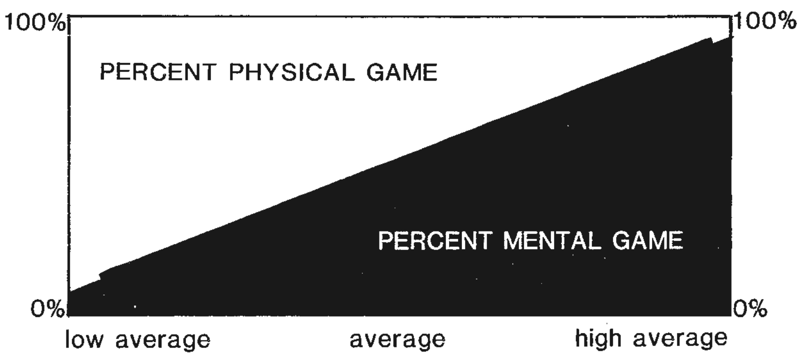


Figure 3. "As your skill increases, the mental game becomes the largest percentage of your game." - George Allen

According to Allen, "As your skill increases, The Mental Game becomes the largest percentage of your game. More correctly, to increase your average to the highest possible level, you must develop your mental game. At the top of successful performance, 90% of the game is mental." Allen goes on to describe two levels of the mental game - the conscious and the subconscious. The conscious mind makes decisions about such things as what ball to use, what part of the lane to play, what release to use, and so on. The subconscious mind is responsible for performing the task for which it was trained.

The intent of this paper is not to explore the mental game of bowling, but to explore the many ways in which mathematics is involved in the game. Figure 3 actually illustrates the first example of the occurrence of mathematics in bowling. What follows is a collection of mathematical applications in bowling. Some may be of more use to the serious bowler, while others will be of more use in the classroom.

1. PROBLEM SOLVING, PUZZLES, AND RECREATIONS

Keeping score in bowling is an excellent application of addition that can be used in grades K-8 and in general math. Since that topic is fairly well known and occurs in many mathematics texts, it will not be covered here. But, after learning to keep score, there are many interesting puzzles or problems that can be considered. In fact, Example 1 occurred in the January, 1984 issue of The Mathematics Teacher.

Example 1. In bowling, a perfect score of 300 can be obtained in only one way: getting twelve strikes in a row. Can any other scores be obtained in only one way?
Answer: Yes, games of 291-299 can be scored in only one way, by getting the first 11 strikes in a row, and then getting 1 to 9 pins on the last ball.

There are many other related problems of interest.

Example 2. How many ways can a 290 game be scored?
Answer: Two. The first way is by getting a spare in the first frame and the next eleven strikes in a row. The second is by getting the first eleven strikes in a row and throwing a gutter ball on the last ball. This has happened - probably a result of a poor mental game on the part of the bowler.

Not all the answers to the following will be given, so you will be challenged.

Example 3. How many ways can a 280 game be scored?

Example 4. In how many ways can each of the games from 270-279 be scored?

The following problems involve some estimating.

Example 5. A game with a double (two strikes in a row) and the rest spares usually results in a game in what range of scores? This assumes that the bowler scores well on the first ball of each spare.
Answer: 200-209

Example 6. A game with a turkey (three strikes in a row) and the rest spares usually results in a game in what range of scores?
Answer: 210-219

You can keep going with this, assuming four, five, six strikes, and so on.

Often when I bowl, and I am in the sixth frame, I would like an estimate of how I am doing. For example, can I obtain "par" - a 200 game?

Example 7. In the sixth frame, what does the tens digit tell you about your score, assuming you will get marks (strikes or spares) for the rest of the game?
Answer: If the tens digit is 0 you are working on a game in the 180s. If the tens digit is 1, you are working on a game in the 190s. If the tens digit is 2, you are working on a game in the 200s. The 2 in the tens digit corresponds to a 2-hundred game. This can be extended - when you add a tens digit, you add 10 to the possible score of your game.

The following more demanding problem appeared in the December, 1982 edition of the Bowler's Journal. It had been in a recent Games Magazine entitled "Alley 1." This puzzle generated a great deal of interest. Free subscriptions were given away for the first five correct solutions. The fact that the magazine received more than 400 correct answers was further evidence to me that bowlers are quite smart. There is only one correct answer. This journal may print the answer in a later issue.

Example 8. Art, Bob, Carl, Denny, and Fred are on the same bowling team. They are all being factual in the following statements regarding the last game they bowled that night.

Art: My score was a prime number. Fred finished third.
Bob: None of us bowled a score over 200.
Carl: Art beat me by exactly 23 pins. Denny's score was divisible by 10.
Denny: The sum of our five scores was exactly 885 pins. Bob's score was divisible by 8.
Fred: Art beat Bob by less than 10 pins. Denny beat Bob by exactly 14 pins.

Determine the score of each bowler in the game.

2. STATISTICS

There are many interesting things one can do with the statistics of bowling scores and a home computer. Each week, after league bowling, I record my scores on a diskette in a program, together with other information such as what ball I used, what lane I bowled on, and what part of the lane I bowled on, and so on. Such information is useful when bowling again on that pair of lanes. Of course, one can compute the mean (μ) each week, but a statistic not usually considered in bowling circles is the standard deviation (σ). At a certain point my stats were

$$\mu = 193.8 \quad \text{and} \quad \sigma = 26.2.$$

At the point in a statistics course where normal distributions are studied one might compute the standard unit normal distribution given by

$$Z = (X - \mu)/\sigma ,$$

where X represents the bowler's scores. Then one can use the tables for a standard normal distribution to make predictions. For example, what is the probability of bowling a game of 290 or better? For $\mu = 193.8$ and $\sigma = 26.2$, the probability of bowling a score of 290 or higher is given by

$$P[X \geq 290] = P[Z \geq (290 - 193.8)/26.2] = P[Z \geq 3.6654]$$
$$= 0.000125 = 1/8000.$$

Therefore, my chances of bowling a game of 290 or higher are about 1 in 8000. Recently, I may have had my big chance, rolling a 299 game. Even if I bowled 20 games a week, I wouldn't have this opportunity again for eight years.

One who enjoys such meanderings through statistics can push things to a further extreme. A computer program was prepared to accommodate the appending of data each week after my league play. It computes averages, standard deviations, by 1st, 2nd, 3rd game bowled. The following were averages after 21 weeks of bowling:

1st Game (μ)	2nd Game (μ)	3rd Game (μ)
193.826	190.739	195.87

It is folly to make predictions about one's future scores, but the program was designed to compute a Least Squares (LS) Line of Best Fit for each future game. But, if you enjoy math, it is fun. Here are the predicted scores for games the next time I bowl:

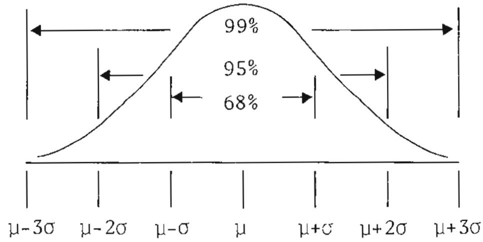
Least Square Predictions

1st Game	2nd Game	3rd Game
197.964	184.81	190.261

One can see some pattern in these results. The slope of the LS line is negative for the second game. I'm slumping on my second games. But, scores on my first game look optimistic.

Another interesting question arises concerning standard deviation. We know that averages of pros are higher than those of most amateurs. But, what about the standard deviations of the pros? Recall that standard deviations give a measure of the "spread" of data about the mean in a distribution. This is illustrated in Figure 4.

(a)



(b)

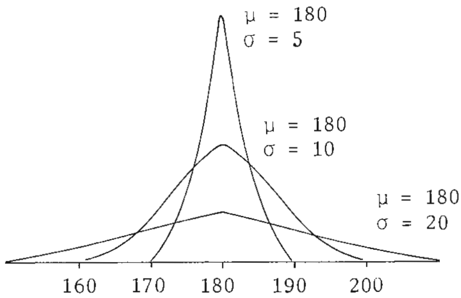


Figure 4. The standard deviation indicates the "spread" about the mean.

My hypothesis was that the standard deviations of pros are higher than the standard deviations of amateurs. To pursue this question I gathered data on professional and amateur bowlers over a period of time. The data on the pros was gathered from sources in various tournaments (1983,4 Peoria; 1983 Denver; 1984 U.S. Open) and does not represent an entire year on a PBA Tour. Virtually all the statistics were based on 32 games or more. The data are shown in the following table. The table also shows each bowler's chances of scoring a game of 290 or higher.

Bowlers	μ	σ	1/n Represents 1 chance in n of scoring 290 or higher
<u>Pros</u>			
Earl Anthony	229.8	29.75	1/46
Mike Aulby	203.6	28.05	1/719
Dave Husted	210.3	27.49	1/518
Randy Stoughton	207.7	28.59	1/503
Gary Dickinson	202.3	34.30	1/191
Steve Cook	213.6	29.61	1/202
Bob Handley	207.8	29.27	1/403
Wayne Webb	205.8	28.99	1/535
Tom Milton	209.1	32.84	1/144
Marshall Holman	207.9	24.15	1/571
Mark Roth	213.5	25.32	1/901

Amateurs

MB	193.8	26.2	1/8000
DW	204.5	29.4	1/535
JB	176.2	16.9	1/100,000
RG	196.5	33.5	1/376
BC	186.9	27.6	1/10,695
GK	201.8	29.5	1/784
CA	178.9	21.5	1/100,000
OF	171.6	20.3	1/100,000
SF	211.4	28.2	1/382
FR	165.8	23.9	1/16,667
DH	156.7	21.6	1/100,000

Using a Wilcoxon Distribution Free Rank Sum Test, the hypothesis that the standard deviations of the pros is higher than that of the amateurs was accepted at the 5% level. On a more common-sense level, one might argue that we can just "see" this result in the table. Also, keep in mind that pros depend much more on strings of strikes for their high averages. When they miss on a string it costs them, say, 10-22 pins in a particular frame, while a low average bowler may not get a string of strikes at all in a game. Thus, there is more variance in the pros' scores.

One last use of standard deviation might be in the selection of a bowler for an amateur tournament. Such tournaments are usually handicap (pins are added to your score based on your average - say, 80% of the difference of your average and 200). If it were possible to know averages and standard deviations of bowlers, you might pick a bowler with a lower average (to get a larger handicap), but with a high standard deviation to take advantage of the fact that this bowler has a higher probability of scoring a high game. It is true that this bowler also has a high probability of bowling a low game, but the risk might pay off.

3. SPARE CONVERSION SYSTEMS

Certain mathematical spare systems have prevailed in the game for some time. As I have bowled the past few years, I have doubted some aspects of the thinking about those systems. What will be done now is analyze these systems in detail. Before doing that, we must establish some features of a bowling lane. Figure 5 shows some aspects of a bowling lane (not drawn to scale for reasons of space).

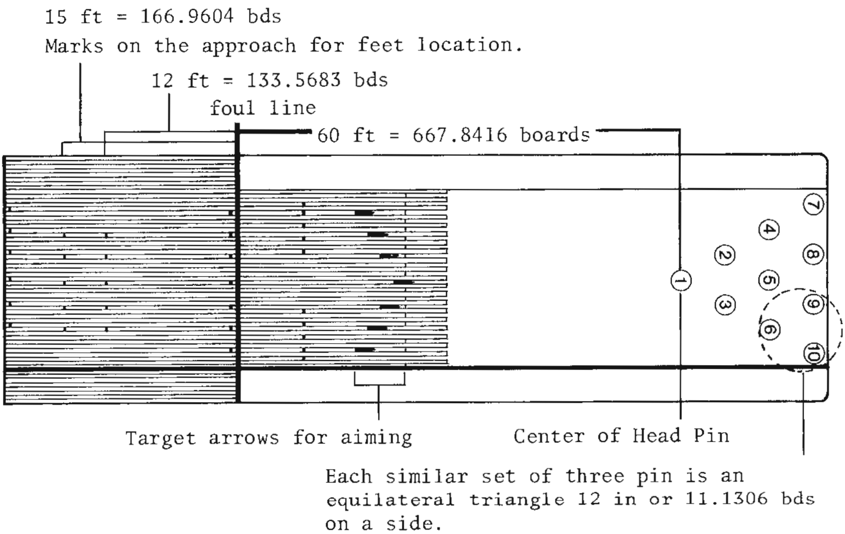


Figure 5. The bowling lane. Not to scale.

A bowling lane is about 41.5 inches wide, and is made up of about 39 boards, each 1.0781 inches wide. From the foul line to the center of the 1-pin, or headpin, is a length of 60 ft, or 667.8416 bds. Note that I am intentionally converting all units to <u>boards</u> because this is what the bowler sees at his feet and out on the lane when he is aiming, and it is common to speak of "how many boards your feet have moved, and how many boards the ball moved." Using "boards" for units also has the advantage of providing a common unit of measure. There are markers on the approach 12 ft, or 133.5683 bds back from the foul line, and at 15 ft, or 166.9604 bds. Seven target arrows, 5 bds apart, occur at approximately 15 ft out on the lane from the foul line, but the fact that <u>none</u> of these arrows is actually at 15 ft is cause of possible error in the use of some spare conversion systems. The distances of these arrows from the foul line is given below.

Middle or 4th arrow	= 15 ft, 4 in.	= 170.6706 bds
3rd arrow	= 14 ft, 10 in.	= 165.1053 bds
2nd arrow	= 14 ft, 6 in.	= 159.5399 bds
1st arrow	= 13 ft, 10 in.	= 153.9746 bds.

Another cause of error in some spare conversion systems is the fact that bowlers line up at different places on the approach. For example, some line up at the 12 ft markers, others at the 15 ft markers, and still others line up elsewhere.

The pins form an equilateral triangle, with each pin 12 inches, or 11.1306 bds, from any it is next to. Related horizontal and vertical distances are given in Figure 6.

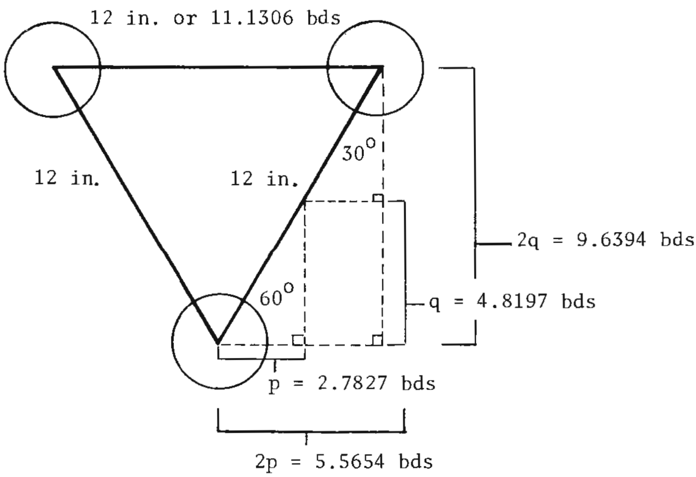


Figure 6. Neighboring pins from equilateral triangles 12 in. on center.

<u>The 3-6-9 Spare Conversion System</u>. Suppose you are throwing your strike ball at a certain target arrow, but you leave a spare like any of those in Figure 7.

Note that the pin you must hit to convert the spare is the 2-pin. It is the "key" pin. To convert this spare using the 3-6-9 spare conversion system you move 3 boards to the right and aim at the same target arrow.

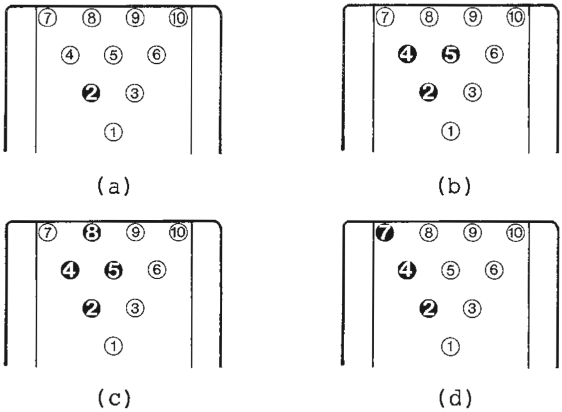


Figure 7. The darkened pins still stand. The "key" pin in each of these spares is the 2-pin.

Suppose you are throwing your strike ball at a certain arrow, but you leave a spare like one of those in Figure 8.

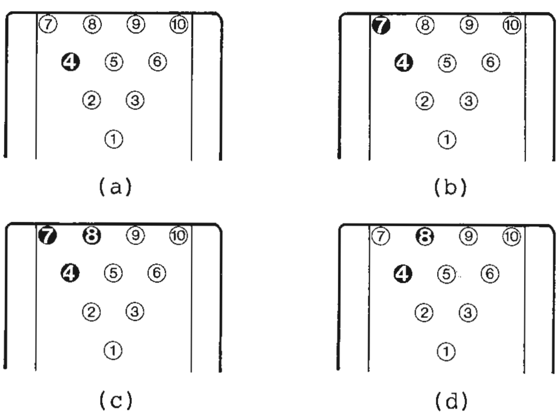


Figure 8. The key pin in each of these spares in this 4-pin.

The key pin you must hit to convert the spare is the 4-pin. To convert this spare using the 3-6-9 conversion system you move 6 boards to the right and aim at the same target arrow.

Suppose you are throwing your strike ball at a certain arrow, but you leave the 7-pin spare shown in Figure 9.

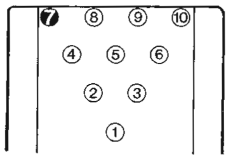


Figure 9. To convert this spare using the 3-6-9 system, move right 9 boards and aim at the same target arrow.

To convert this spare using the 3-6-9 conversion system, you move 9 boards to the right and aim at the same target arrow.

The preceding discussion applied to right-handed bowlers. Left-handed bowlers would apply this system when the key pins in their spares are the 3-pin, 6-pin, and the 10-pin.

To evaluate the appropriateness of this system we form a mathematical model of the path of the ball and apply some analytic geometry and trigonometry.

We coordinatize the entire lane by placing an x-axis along the right-hand edge of the lane and a y-axis at the foul line with the origin as shown in Figure 10. Coordinates of certain locations and pins are also indicated in Figure 10.

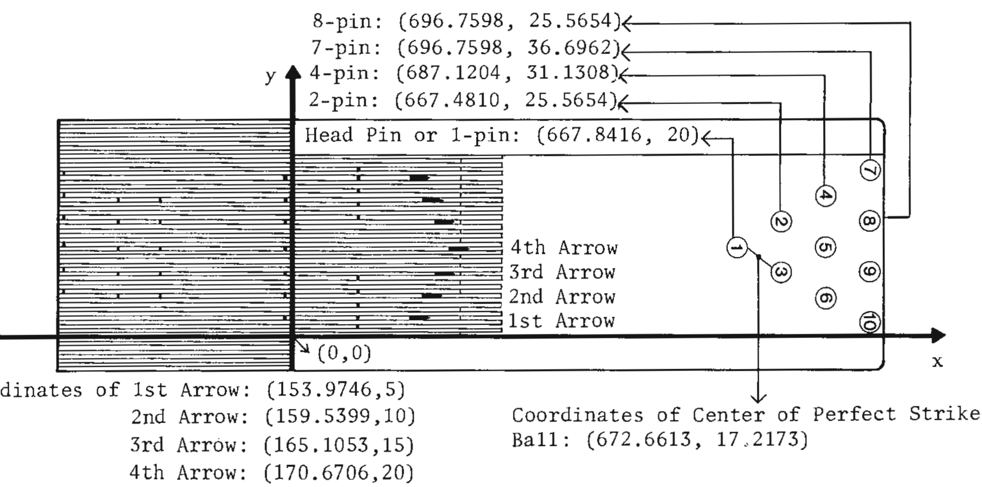


Figure 10. Carterian coordinates for a bowling lane.

(Left handers can follow this discussion by considering the x-axis on the left side of the lane and the positive y-axis pointing to the right. Coordinates listed earlier are then the same).

The actual flight of a typically-thrown bowling ball varies somewhere between a straight line and a large arc or curve. In the 3-6-9 spare system one aims at the same target for the strike ball, and having moved a certain number of boards to the right, looks again at the same target. Thus we can think of the line of flight of the ball being pivoted at the target. Suppose (Figure 11) that the target is the 2nd arrow (point P). The strike ball is thrown along a line from point R to P and along an arc from P to S, where S is the location of the center of a perfect strike ball. We model the arc by the line PS of length E. Now,

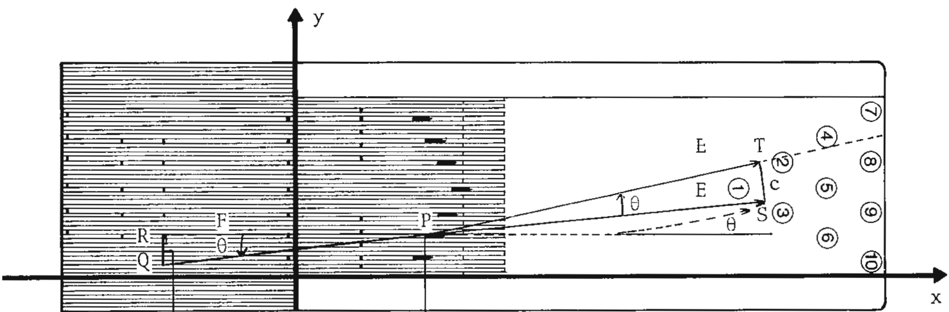


Figure 11. Target: the second arrow.

F = length of RP = Distance from starting point R (front of foot) to the foul line + Distance from foul line to target arrow.

E = Distance from target arrow P to S. We find E using the given coordinates of P and S and the distance formula.

Thus, the path of the ball is modeled by the union of two lines (an "elbow"), that meet at the target arrow, or pivot point. After a move of A boards to the right, your shoulder and arm have been rotated through an angle given by

$$\theta = \arctan(A/F).$$

After the rotation, the point S of the perfect strike is moved a distance c to a new point T, where c is given by

$$c = E \cdot \sin\theta / \sin(\pi - \theta)/2 .$$

The formula for c is found using the laws of sines. We assume that the coordinates of the of the pivot point P are (X_1, Y_1). The coordinates of point T are (X_2, Y_2).

Now,

$$X_2 = E\cos(\theta + \phi) + X ,$$

and

$$Y_2 = E\sin(\theta + \phi) + Y ,$$

where

ϕ = the angle between the line PS and the horizontal
  $= \arctan (Y_1 - 17.2173)/(X_1 - 672.661)$.

After we make this move of A boards to convert a spare, we are interested in the flight of the ball through point T and beyond to the pins. We can use the two-point equation of the line since we now have coordinates of P and of T. The line has the following equation

$$Y = \left[(Y_2 - Y_1)/(X_2 - X_1)\right] \cdot (X - X_1) + Y_1 .$$

If we substitute the first coordinate of each key pin into this equation, it will tell us where the ball is at that point where the ball is vertically aligned with the pin. (See Figure 12.) Making these computations for certain starting positions will allow us to validate the spare conversion system.

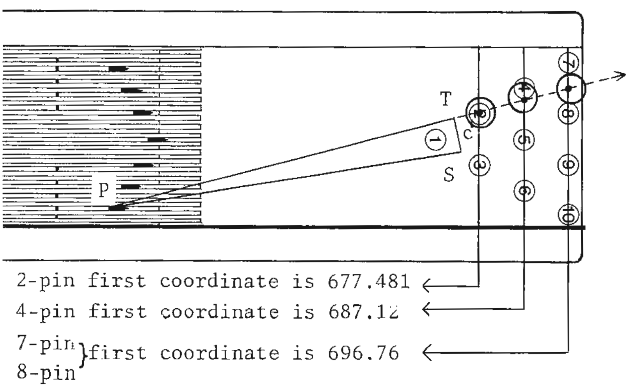


Figure 12.

Target: Second Arrow. Let us make some computations. (Actually a computer program was used.) Suppose the starting point is at the 12 ft markers on the approach and the pivot point or target arrow is the 2nd arrow. Then

$$X_1 = 159.94, \qquad Y_1 = 10$$

$$E = 513.172, \qquad F = 293.108.$$

From the computer program and the formulas given we obtain the following table of values.

TABLE 1 (2nd arrow, 12 ft approach markers, pivot at target arrow)

A(boards moved on approach) Radius of ball = 4.08 bds
C(boards moved at pin deck=ST) Radius of pin = 2.2 bds
Y(second coordinate of center of ball on the projected line - occurs when ball is at the 1st coordinate of the given pin)

A	c	θ(degrees)	Y(2-pin)	Y(4-pin)	Y(7-pin)
1	1.75	0.195	19.05	19.22	19.39
2	3.5	0.391	20.82	21.02	21.22
3	5.25	0.586	22.59	22.82	23.06
4	7.00	0.782	24.36	24.62	24.89
5	8.75	0.977	26.12	26.42	26.73
6	10.50	1.173	27.89	28.23	28.56
7	12.25	1.368	29.66	30.03	30.39
8	14.00	1.563	31.43	31.83	32.23
9	15.75	1.759	33.20	33.63	34.06
10	17.50	1.954	34.97	35.43	35.90
11	19.25	2.149	36.74	37.23	37.73
Actual values of the second coordinate of pin			25.56	31.13	36.70

Suppose the starting point is changed to the 15 ft markers. Then F becomes 326.5 and the table is changed as follows.

Table 2 (2nd arrow, 15 ft markers, pivot at the target)

A	c	0(degrees)	Y(2-pin)	Y(4-pin)	Y(7-pin)
1	1.57	0.175	18.87	19.04	19.20
2	3.14	0.351	20.46	20.65	20.85
3	4.72	0.526	22.05	22.27	22.49
4	6.29	0.702	23.63	23.89	24.14
5	7.86	0.877	25.22	25.50	25.79
6	9.43	1.052	26.81	27.12	27.43
7	11.00	1.228	28.40	28.73	29.08
8	12.57	1.404	29.98	30.35	30.73
9	14.14	1.579	31.57	31.97	32.37
10	15.71	1.754	33.16	33.59	34.02
11	17.28	1.930	34.49	35.21	35.67
12	18.85	2.105	36.33	36.82	37.32

Let us begin to make some analysis. Note in Table 1 that a 1-board move on the approach (A = 1) results in a 1.75-board move at the pin deck (c = 1.75). This difference holds through the table; each time you move your feet 1 bd, you get a 1.75-board reaction at the pin deck. But, if you start at the 15 ft markers, as in Table 2, a 1-board move in the approach (A = 1) results in a 1.57-board reaction at the pin deck (c = 1.57). Thus, we have at least shown (and will continue to show) that different starting positions at the markers results in different locations of the ball at the pin deck.

Let us check to see if each of the 3-6-9 moves will allow the ball to hit the 2-pin, 4-pin, and 7-pin, respectively. Under column A we find 3, for a 3-board move of the feet. We move across Table 1 under the 2-pin column and find that the center of the ball has a second coordinate of 22.59 when it is aligned vertically with the 2-pin. Then we add and subtract the radius of the ball:

Actual location of the center of the ball + radius of the ball
= 22.59 + 4.08
= 26.67

Actual location of the center of the ball - radius of the ball
= 22.59 - 4.08
= 18.51.

Thus, the vertical interval covered by the ball is [18.51,26.67].

The actual second coordinate of the center of the 2-pin is 25.56. We add and subtract the radius of the pin, 2.2 bds.

Location of center of 2-pin + radius of pin
= 25.56 + 2.2 = 27.36

Location of center of 2-pin - radius of pin
= 25.56 - 2.2 = 23.36.

Thus, the interval covered by the 2-pin is [23.76,27.36]. Since the two intervals overlap (Figure 13) the 2-pin is covered and will be converted.

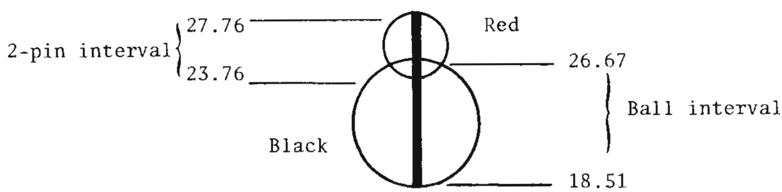


Figure 13.

Suppose (Table 1) that you made a 6-board move to the right in order to make a spare where the key pin is the 4-pin (Figure 8). Look across the table from A = 6 under Y(4-pin) and you find that the center of the ball has a second coordinate of 28.23 when it is aligned vertically with the 4-pin. Now the actual second coordinate of the center of the 4-pin is 31.13. Then we add and subtract the radius of the ball:

Actual location of the center of the ball
+ radius of the ball
= 28.23 + 4.08
= 32.31

Actual location of the center of the ball
- radius of the ball
= 28.23 - 4.08
= 24.15.

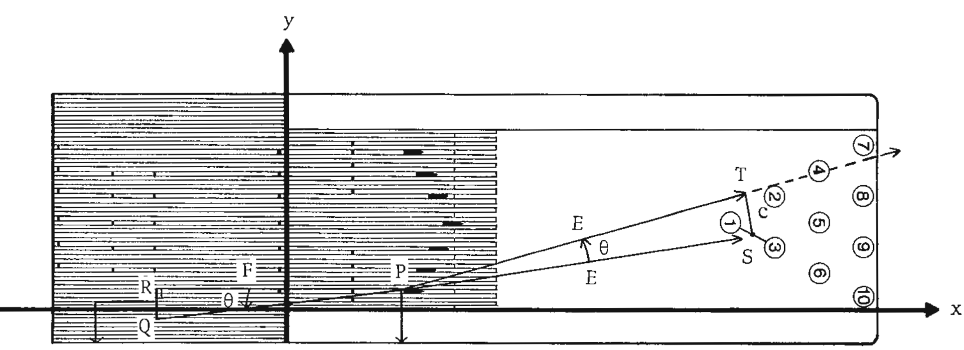
Thus, the interval covered by the ball is 24.15,32.31

The actual second coordinate of the center of the 4-pin is 31.13. We add and subtract the radius of the pin, 2.2 bds, and we get the interval covered by the 4-pin to be [28.93,33.33]. Since the two intervals overlap, the 4-pin will be converted.

It will be left to the reader to verify that a 9-board move will indeed meet the goal of converting the 7-pin. The interval covered by the 7-pin is [34.50,38.90]. We will use the pin intervals again.

Note, however, that if your goal was to have the center of the ball go through the center of the pin it would have been better to move about 4½ boards for the 2 pin. Look down the 2-pin column until you get close to the actual value 25.56, then look to the left to find the number of boards which would have to be moved, about 4½. Similarly, an 8-board move is better for the 4-pin, and an 11-board move is better for the 7-pin. Thus, if your target is the second arrow and you start at the 12 ft markers, the 3-6-9 system might be improved if it were 4½-8-11.

We leave it to you to verify that the 3-6-9 system will work from the 15 ft markers if the target is the second arrow (See Table 2), but it might be improved if it were 5-9-11.



Movement of feet
A boards to the right

Pivot point: 1st Arrow in this case

Figure 14. Target: The first arrow.

Target: First Arrow. Now let us change the target arrow to the first arrow. See Figure 14.

Suppose the starting point is at the 12 ft markers on the approach. Then

$$X_1 = 153.975, \qquad Y_1 = 5$$

$$E = 518.831, \qquad F = 287.543.$$

From the computer program and the formulas given we obtain the following table of values.

TABLE 3 (1st arrow, 12 ft marker, pivot at the target)

A (boards moved on the approach) Radius of ball = 4.08 bds
c (boards moved at pin deck) Radius of pin = 2.2 bds
Y (second coordinate of center of ball on the projected line-occurs when ball is at 1st coordinate of the given point)

A	c	θ(degrees)	Y(2-pin)	Y(4-pin)	Y(7-pin)
		(Actual values	25.56	31.13	36.70)
1	1.8	0.199	19.15	19.41	19.67
2	3.6	0.399	20.82	21.27	21.56
3	5.4	0.598	22.80	23.12	23.45
4	7.2	0.797	24.62	24.98	25.34
5	9.0	0.996	26.44	26.84	27.23
6	10.8	1.195	28.26	28.69	29.12
7	12.6	1.395	30.09	30.55	31.01
8	14.4	1.594	31.91	32.41	32.90
9	16.2	1.793	33.74	34.27	34.80
10	18.0	1.992	35.56	36.12	36.69

Suppose the starting point is changed to the 15 ft markers. Then F is changed to 320.935 and the table is changed as follows.

TABLE 4 (1st arrow, 15 ft marker, pivot at the target)

A	c	0(degrees)	Y(2-pin)	Y(4-pin)	Y(7-pin)
1	1.6	0.179	18.96	19.22	19.48
2	3.2	0.357	20.60	20.88	21.17
3	4.8	0.535	22.23	22.55	22.86
4	6.5	0.714	23.86	24.21	24.56
5	8.1	0.893	25.49	25.87	26.25
6	9.7	1.071	27.13	27.54	27.94
7	11.3	1.250	28.76	29.20	29.64
8	12.9	1.428	30.40	30.86	31.33
9	14.5	1.606	32.03	32.53	33.02
10	16.2	1.785	33.66	34.19	34.72

We leave it to you to verify that the 3-6-9 system does indeed hold for a shot at the first arrow from either the 12 ft or 15 ft markers.

<u>One word of caution</u>! There are many spares, such as the 2,4,5 in Figure 7(b), that should not be covered by having the center of the ball go through the center of the 2-pin. A 4½-board move works best for getting the center of the ball to go through the center of the 2-pin from the second arrow at 12 ft, but a 4-board move should work better to convert the 2,4,5 spare as in Figure 7(b).

<u>Other words of caution are in order</u>. The intent here is to document some theories of spare conversion, but the bowler has to fit this to his game, and always keep in mind other aspects of the game. For example, bowlers often find more oil or lane conditioner in the center of the lane. Then an extra board move to the right might be necessary. On the other hand, sometimes there is less oil in the center of the lane. Then the bowler might move one less board. Also keep in mind that different bowlers put different amounts of hook, turn, revolutions, and speed on the ball and this has an effect on the roll of the ball across the pin deck. Even the same bowler might not throw the same ball on a spare ball as a strike ball.

The verification of the 3-6-9 system can be continued for shots from the 3rd arrow, 4th arrow, and so on, from any location on the approach to any target near the arrows. But, the tables are lengthy. A computer program for the construction of these tables is given at the end of this article.

In summary, the 3-6-9 system is valid, but it needs to be "personalized," so to speak, by the bowler and does indeed depend on where the bowler stands on the approach and the target at the arrows. For example, from the 15 ft marker a shot at the 3rd or 4th arrows after a move of 9 boards, will hit the 7-pin but very lightly. A 12-board move would be more appropriate. Table 5 summarizes how the 3-6-9 system might be revised to guide the center of the ball through the center of the pin.

TABLE 5 3-6-9 Revised

Target Arrow	Approach Location	Adjustment System
1st	12 ft	4½-7½-10
	15 ft	5- 8-11
2nd	12 ft	4½-8½-11
	15 ft	5- 9-11
3rd	12 ft	5- 8-11
	15 ft	5½- 9-12
4th	12 ft	5-8½-11
	15 ft	5- 9-12

Professional Bowling Association member Mike Durbin lends some practical credence to the preceding modified system. He has suggested in his TV tips that "the 5-8-11 spare system works well for me." Indeed, Durbin starts well back on the approach.

The 3-1-2 Spare System. Rather than a complete spare system the 3-1-2 refers to the more simple idea that if you move your feet two boards to the right on the approach and release the ball one board to the right at the foul line, and maintain your same target arrow, the ball will be moved three boards to the left at the pin deck. This rule CANNOT be taken as a blanket statement! Check Tables 1 through 4. Data from these tables, and others constructed from the computer program, are given in Table 5A below.

TABLE 5A Board Moves

Target Arrow	Approach Location	Number of Boards Ball is Moved at Pin Deck
1st	12 ft	3.6
	15 ft	3.23
2nd	12 ft	3.5
	15 ft	3.14
3rd	12 ft	3.4
	15 ft	3.05
4th	12 ft	3.3
	15 ft	2.97

Note that the system does hold fairly well if you are standing at the 15 ft markers.

In summary, the 3-1-2 system does hold, but only if you are well back on the approach.

The 2-4-6 system. In the 3-6-9 system you move your feet and cross the same target arrow. In effect, you pivot your path to the ball at the target. You visually face a different target and pivot at your feet. For example, to convert the 2-pin, you stand in the same place, but pivot your feet in such a way to move your target board two boards to the left. In bowling lingo, "you move your eyes." To convert a 4-pin spare you stand in the same place, but pivot your feet to move your target board four boards to the left. Similarly, to convert the 7-pin you stand in the same place, but pivot your feet to move your target board six boards to the left. This is illustrated in Figure 15.

Interestingly, triangle PQR of Figure 14 gets flipped in Figure 15, and the mathematical formulas and computer program can still be used. One just has to keep in mind that E now stands for the total distance from the feet (not the target arrow) to the location of the perfect strike ball, F is the same as before, X_1 is now the first coordinate of your feet or pivot, and Y_1 is the second coordinate.

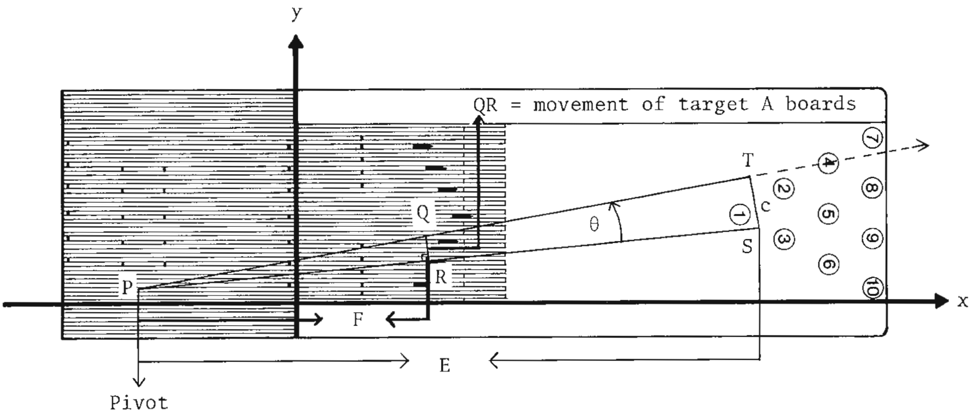


Figure 15. In the 2-4-6 system, you pivot your feet.

Let us first consider the first arrow from the 12 ft markers. Then

$$X_1 = -133.568, \qquad Y_1 = 5,$$

$$E = 806.374, \quad \text{and} \quad F = 287.543.$$

From the computer program and the formulas given we obtain the following table.

TABLE 6 (1st arrow, 12 ft markers, pivot at the feet)

A	c	θ(degrees)	Y(2-pin)	Y(4-pin)	Y(7-pin)
1	2.80	0.199	20.11	20.29	20.47
2	5.61	0.399	22.93	23.15	23.36
3	8.4	0.598	25.63	26.00	26.25
4	11.23	0.797	28.58	28.86	29.14
5	14.02	0.996	31.40	31.71	32.03
6	16.82	1.195	34.22	34.57	34.92
7	19.62	1.395	37.04	37.43	37.81
8	22.43	1.594	39.87	40.28	40.70

As before, we check to see if each move will allow the ball to hit the pin. Under column A we find 2 for a

2-board move of the target. We move across under the 2-pin column and find 22.93. Then we add and subtract the radius of the ball:

Actual location of center of ball + radius of ball
= 22.93 + 4.08 = 26.98

Actual location of center of ball - radius of ball
= 22.98 - 4.08 = 18.90.

The interval covered by the 2-pin is [23.76,27.36], so the 2-pin is covered.

Similarly, on a 4-board move with the target, the interval covered by the ball is [24.81,32.97], and the interval covered by the 4-pin is [28.93,33.33]. The intervals overlap, so the 4-pin is covered or converted.

For a 6-board move the interval covered by the ball is [30.84,39.00]. The interval covered by the 7-pin is [34.50,38.90]. The intervals overlap, so the 7-pin is covered.

The 2-4-6 system is often proposed for an outside shot such as this, but if you look carefully at TABLE 6, you will see that if you want to hit the 2-pin more directly a 3-board move would have been better from the 12 ft marker. For the 4-pin, a 5-board move would have been almost perfect, and for the 7-pin a 7-board move would have been closer. Though we will not take the space for another table, this also holds true for the 15 ft markers.

<u>In fact, from any of the 1st, 2nd, 3rd, 4th arrows, and from either the 12 ft or 15 ft markers, the 2-4-6 system will work for spare conversion, BUT the 3-5-7 system will make for a more direct hit of the key pins</u>.

Again, keep in mind that you will have to adapt any of this information to your own game.

One final comment about the distance F in the model for the path of the ball. It might be argued that the pin deck results would be off should you be starting your strike ball from a position other than straight back from your target. In fact, as much as a 10-board move in your starting position results in only a 0.01 board change in the length of F. This can be checked with the Pythagorean Theorem.

4. CONCLUSION, SOME HOMEWORK, AND A COMPUTER PROGRAM

We have considered many applications of mathematics in bowling. Some are more for fun and recreation while the spare conversion topic is more for the serious bowler.

Homework Problems:

1. What column in the Tables would you check to convert the 8-pin? (The answer is NOT the 2-pin column.)

2. Wayne Webb, a professional bowler, starts about 10½ feet back from the foul line. Design a "Move Your Feet" spare system for Wayne Webb.

The following BASIC computer program was used for the spare computations in this article. Line 100 and line 739 can easily be adjusted for moves of more than 10 boards.

```
1 INPUT E, F, X1, Y1
2 LPRINT "E=distance from pivot point to strike target,
  in boards=";E
3 PRINT  "E=distance from pivot point to strike target,
  in boards=";E
4 LPRINT "F=distance from feet to target arrow, in
  boards=";F
5 PRINT  "F=distance from feet to target arrow, in
  boards=";F
6 LPRINT "X1=first coordinate of pivot point ="; X1
7 PRINT  "X1=first coordinate of pivot point ="; X1
8 LPRINT "Y1=second coordinate of pivot point =";Y1
9 PRINT  "Y1=second coordinate of pivot point =";Y1
10 LPRINT "A=number of boards moved"
12 PRINT  "A=number of boards moved"
13 LPRINT "C=number of boards moved at the pin deck"
14 PRINT  "C=number of boards moved at the pin deck"
15 LPRINT "G=angle, in degrees, shoulder and body moved"
16 PRINT  "G=angle, in degrees, shoulder and body moved"
80 LPRINT "A "; "  C    "; "   G      "; "   X2   "; "
   Y2   "; "      P2";"P4"; "        P7"
90 PRINT "A  "; "  C    "; "  G      "; "    X2    "; "
   Y2   "; "      P2";"P4"; "P7"
100 FOR I=1 TO 10
200 READ A
```

```
300 D=ATN(A/F)
400 G=D 180/3.141592654#
500 C=E SIN(D)/SIN((3.141592654#-D)/2)
510 H=ATN((Y1-17.2173)/(X1-672.661))
520 X2=E COS(D+H)+X1
530 Y2=E SIN(D+H)+Y1
540 P2=((Y2-Y1)/(X2-X1)) (677.481-X1)+Y1
550 P4=((Y2-Y1)/(X2-X1)) (687.12-X1)+Y1
560 P7=((Y2-Y1)/(X2-X1)) (696.76-X1)+Y1
600 LPRINT A; C; G; X2; Y2; P2; P4; P7
700 PRINT  A; C; G; X2; Y2; P2; P4; P7
710 NEXT
720 END
739 DATA, 1,2,3,4,5,6,7,8,9,10
```

About the Author

The author of more than sixty mathematics textbooks, Professor Bittinger also has bowling as an avocation, participating not only in league play, but many amateur and professional tournaments. His love of the game has prompted the writing of this paper where the rich interaction of mathematics with the game of bowling is brought to light.